A Note on Core Regions of Membership Functions

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ABSTRACT: In neuro-fuzzy approaches different membership functions are used for modeling the system's rule set. Two wellknown membership function types are triangle functions and trapezoid functions. In our contribution we demonstrate that trapezoid functions with larger core regions are the more appropriate functions for calculating the membership degrees within neuro-fuzzy systems. If regions of the data of different classes are highly overlapping or if the data is noisy, the values of the membership degrees could be misleading with respect to rule confidence if the core region is modeled too small. In fact, we show that data regions with a high membership degree need not to be the regions with a high rule confidence. This effect that we call membership unrobustness is discussed. We give preliminary benchmark examples and show how this effect influenced our recent work of analysing septic shock patient data.

KEYWORDS: neuro-fuzzy system, membership function, core region, rule confidence, robustness, medical data

INTRODUCTION

An up to date neuro-fuzzy system is the so-called Fuzzy-RecBF-DDA [1], [2], that we recently used in an improved implementation for rule generation and classification in the medical domain [3] (analysis of septic shock patient data). The system is based on the dynamic, geometric adaptation of trapezoids. Another wellknown system is NEFCLASS [4] that is based on a fuzzy backpropagation adaptation process. Fuzzy-RecBF-DDA uses trapezoid membership functions. NEFCLASS uses triangle membership functions. We will not discuss the main differences of the two systems and we will not decide which system is the better one. The interested reader should refer to the literature if he/she is interested in the details of the different systems.

Our approach is a general comparison of the main two types of membership functions: the trapezoid and the triangle membership function, mainly their core regions. Usually for adapting neuro-fuzzy systems the data is divided into training and test data. The training data is used to train the system, the test data is used to calculate the performance of the system. If the data is noisy or if the data of different classes is highly overlapping or if there are not much samples for a performant generalization available - an usual szenario in real world applications - then the statistical properties of the training set may differ from those of the test set. We argue that in these situations trapezoid membership functions with larger core regions should be preferred.

In the next section we define the membership functions and explain the training and test procedure. Then, by some examples we show that it is more reasonable to use trapezoid membership functions with larger core regions instead of triangle membership functions. Particularly, we discuss our more fundamental considerations by septic shock patient data that was recorded in intensive care units from 1993 to 1997 at the Klinikum der J.W. Goethe-Universität Frankfurt am Main [5].

PRELIMINARIES

After presenting the definition of some important fuzzy theory terms we define triangle and trapezoid membership functions. Then, we describe how we will divide the data into training and test data for the adaptive phase of a neuro-fuzzy learning.

MEMBERSHIP FUNCTIONS

In Def. 1 we explain the important terms *membership function* and *membership degree*. We define the terms a *-cut*, a *-region* and a *-rule*.

Definition 1:
Let X be a set. A is called a *fuzzy set* if there exist a corresponding *membership function* $m : X \to [0,1]$ that is defined everywhere on X. The value $m(x)$ is called *membership degree* of $x \in X$. The region in the data space where $m(x) = a$ we call a *-cut* if we consider the membership function. The corresponding geometric region we call a *-region*. If additionally the whole rule with the conclusion is considered we speak about a *-rules*. In the special case of $a = 1$, i.e. when considering the 1-cut of a membership function, we speak about *core regions* resp. *core rules*.

In the following example we give the definitions of triangle and trapezoid membership functions in the one-dimensional case, cf. Figure 1.

Example 1:
a) Triangle membership function (1) with $a_1, a_2, b \in I\!R$:

$$m_{triangle}(x) := \begin{cases} 1 & , \quad x = b \\ \dfrac{x - a_1}{b - a_1} & , \quad x \in [a_1, b) \\ \dfrac{a_2 - x}{a_2 - b} & , \quad x \in (b, a_2] \\ 0 & , \quad otherwise \end{cases} \qquad (1)$$

b) Trapezoid membership function (2) with $a_1, a_2, b_{1,} b_2 \in I\!R$:

$$m_{trapezoid}(x) := \begin{cases} 1 & , \quad x \in [b_1, b_2] \\ \dfrac{x - a_1}{b_1 - a_1} & , \quad x \in [a_1, b_1) \\ \dfrac{a_2 - x}{a_2 - b_2} & , \quad x \in (b_2, a_2] \\ 0 & , \quad otherwise \end{cases} \qquad (2)$$

In the m-dimensional case we obtain the membership degree of $x = (x_1, \ldots, x_m)$ by calculating the minimum of the one-dimensional membership degrees of $x_1, \ldots, x_m$ considering the one-dimensional projections of the m-dimensional membership function.

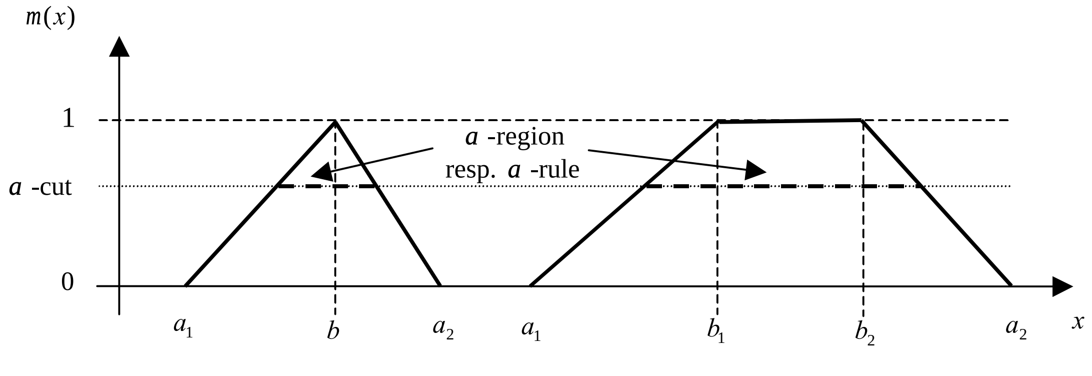

Figure 1: (Left) triangle membership function (Right) trapezoid membership function. The dotted line represents the a-cut. The dashed line in the height of the a-cut represents the a-region resp. the a-rule, the latter if considered with a class label for conclusion.

With d as the dimension of the space the triangle membership functions have $2d$ free parameters. The trapezoid membership functions have $4d$ free parameters. With respect to free parameters the triangle membership functions are the more simpler functions. In the case of $b_1 = b_2$ the trapezoid function is identical to the triangle function.

TRAINING AND TEST

Training a system has to be done carefully. To avoid overfitting the data is usually divided into *training* and *test data*. If we would take all the data for training the system, it would be very well adapted to the training data, but it would not be able to generalize well, i.e. having a good performance on unknown data. Calculating the system's performance on test data is a good way to evaluate a system's generalization ability. Sometimes even more sophisticated data divisions are used (additional validation data, k-fold cross validation, see e.g.[6]).

The statistical properties of the training data differ from those of the test data. If the data environment is noisy or if there are not many samples available the difference is larger. This is the reason why the generalization performance should be calculated only on the test data. If the system is able to tolerate noise and other, different samples then it is well trained and generalizes well.

To calculate the rule performance of an a -rule, i.e. hyper rectangular data region with class label (cf. Def. 1), we define the *rule confidence* as the number of samples of the correct class that lie in the hyper rectangle divided by the number of all the samples that lie in the hyper rectangle. For example, consider the two dimensional unit square. Let there be 4 samples of the class c lying in this square and one of another class $d \neq c$, then the confidence of this a -rule for the class c is $0{,}8$ resp. $80\,\%$.

MEMBERSHIP DEGREES OF TRIANGLE AND TRAPEZOID MEMBERSHIP FUNCTIONS

In this section we present examples which show that it is possible to obtain unexpected results with regard to the rule confidence by using too small core regions together with training and test data.

EXAMPLES FOR “MEMBERSHIP UNROBUSTNESS”

For the first example an artificial dataset is used, motivating the problem of membership unrobustness. For the following examples we use real world datasets to demonstrate the relevance of this problem in practice.
In Example 4 we discuss the problem area by our actual septic shock patient data.

Example 2:
Let us have a more detailed look on the triangle membership function, defined in (1). For easier calculation by hand it is symmetric in our example, i.e. $b - a_1 = a_2 - b$. The upper angle is set to 90°, the lower left and right each to 45°. Thus, the sides of the triangle have a gradient of 1 resp. -1. Let us consider $n_1 = 101$ one-dimensional uniformly distributed samples of the same class c in the interval $[-1,1]$. Add $n_2 = 101$ identical samples with the exception that one sample (the one in the middle, i.e. the one placed at $b = 0$) is a sample of another class d, e.g. an outlier (noise). Let us assume that we have chosen the first $n_1 = 101$ samples for training and the $n_2 = 101$ samples for testing, so that we have obtained the above desribed symmetric triangle membership functions with $a_1 = -1, a_2 = 1, b = 0$.

Now, we calculate the rule confidence for all rectangular regions, considering the membership degrees $0.00;0.02;0.04;\ldots;0.96;0.98;1.00$. Because of the outlier at $b = 0$ the rule confidence for class c for the region with membership degree a (a -rule) is $\frac{100(1-a)}{101-100a}$ %.

In Figure 2 the rule confidence in relation to a is shown. The higher the membership degree is in Figure 2, the lower is the rule confidence that was calculated on the test data. The minor difference in training and test data (one outlier) leads to a result that is surprising. What we expected is an ascent of rule confidence and not a descent. In our example this unwanted effect appears because the core region of the triangle is a single point only. Thus, only one single point in the test set needs to differ and the result is not reliable.

If we use the trapezoid membership function with a core region of a size covering nine test data samples (including the outlier), the rule confidence at membership degree 1 is 88.89 %, the lowest value for the rule confidence considering all membership degrees in this case, but a much better value than in the triangle case (0 %).

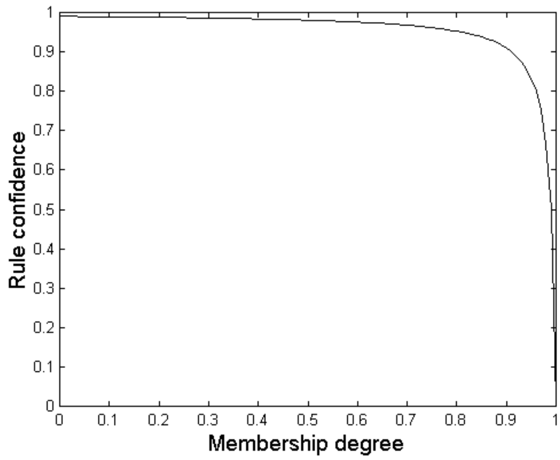

Figure 2: Rule confidence in relation to membership degree. In this example the rule confidence is monotone *descending*.

Now, one could argue that this is a very artificial dataset, but unfortunately the situation remains the same on real world data, see next section. We call this effect *membership unrobustness*.

Example 3:
To demonstrate the effect of membership unrobustness we calculated - in a similar way as in Example 2 - the rule confidence for asymmetric trapezoid membership functions obtained by training the system [3]. As benchmark data we used datasets from the Proben archive [7]. Here, we present the results using the dataset Diabetes1 with overlapping class regions. Approx. 25 % of the data could not be classified correctly. In Figure 3 the rule confidence (mean of all the rules) in relation to the membership degree is shown for both classes of Diabetes1. A higher rule confidence for class one is obtained at lower membership degrees (0.0 to 0.4). The best rule confidence for class two is obtained at the membership degree 0.6 . Thus, the size of the core rules should not be smaller than the size of the rules with these corresponding membership degrees.

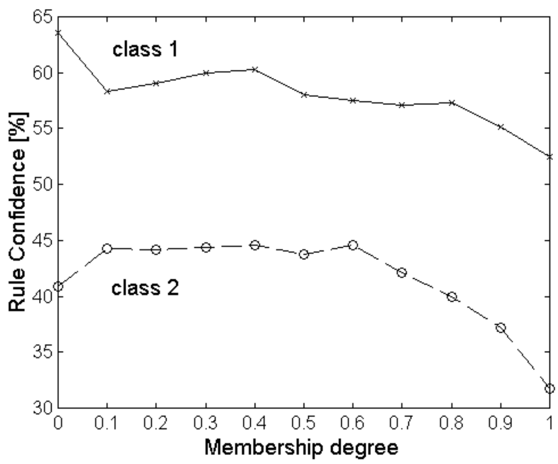

Figure 3: Mean rule confidence (in %) in relation to the membership degree for the two classes (Diabetes1 dataset).

Using too small core regions, the effect of membership unrobustness is very hazardous when cutting a -rules from the trapezoids at higher membership degrees. These a -rules have in fact a lower rule confidence than a -rules of lower membership degrees. Of course, if we would use the triangle membership functions instead of trapezoid membership functions the effect of membership unrobustness would even be stronger since one single point (the top of the triangle) is the smallest not empty core region that is possible.

Example 4:
Let us consider another, more important real world example. In abdominal intensive care medicine patients are in a very critical condition. Often patients develop a septic shock that is associated with a high lethality of about 50 %. It is always related to measurements leaving the normal range (e.g. blood pressure, temperature, respiratory frequency, number of leukocytes), and it is often related to multiorgan failure. We consider 70 patients who developed a septic shock during their stay at the intensive care unit of J.W. Goethe University Clinic. The data was collected from 1993 to 1997 [5]. 38.6 % of the 70 septic shock patients deceased.

To demonstrate the effect of membership unrobustness let us consider the dataset F_{12}, containing the measurements of the 12 most frequently measured variables: creatinin [mg/dl], calcium [mmol/l], arterial pCO_2 [mmHg], pH [-], haematocrit [%], sodium [mmol/l], leukocytes [1000/µl], haemoglobin [g/dl], central venous pressure (CVP) [cmH_2O], temperature [°C], heart frequency [1/min], systolic blood pressure [mmHg]. To limit the influence of missing values, we demanded the existence of a minimum of 10 out of 12 variables for each sample, so that 1698 samples remained out of 2068 (1177 survived, 521 deceased).

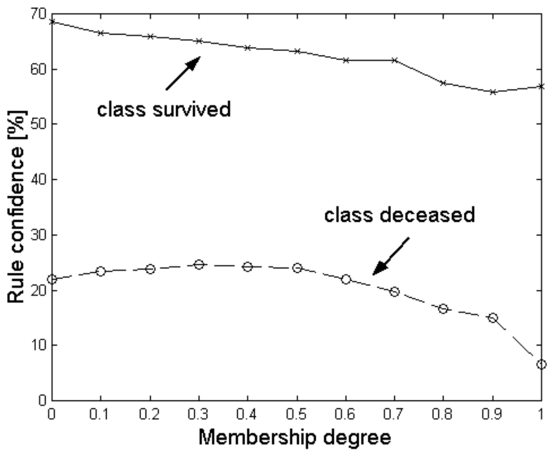

Figure 4: Mean rule confidence (in %) in relation to the membership degree for the two classes (dataset F_{12}).

In Figure 4 we see that the highest rule confidence is obtained at membership degree 0 (0-rules) for class survived and at membership degree 0.3 (0.3-rules) for class deceased. This is another example for membership unrobustness using too small core regions. Since the data probability of class deceased is lower than for class survived, the mean values of rule confidence for class survived are naturally higher than for class deceased.

To get an impression of the results, we present one a-rule for the class survived at membership degree $m = 0.3$ and one at membership degree $m = 1.0$. Rule 1 has a higher confidence on the test data than rule 1*. Thus, rule 1 instead of rule 1* should be presented to a physician. Another benefit of rule 1 is the higher frequency on test data. Due to the lower (statistically too low!) frequency of rule 1* on test data, also the confidence on test data is not as high as for rule 1. In rule 1 resp. 1* the variables systolic blood pressure, arterial pCO_2, haemoglobin, creatinin and sodium are not relevant. Technical note: not all the values of the variables for different membership degrees need to differ (e.g. heart frequency). This is due to algorithm [3] where the trapezoid shape may degenerate to a rectangular shape for some dimensions in the positive and/or negative direction.

Rule 1 ($m = 0.3$):
if var heart frequency ≤ 110.16 **and**
var CVP ≥ 7.00 **and**
var pH ≥ 7.37 **and**
var temperature in $(34.45, 37.42)$ **and**
var leukocytes ≤ 21.50 **and**
var haematocrit ≥ 26.04 **and**
var calcium ≤ 2.50
then class survived **with** frequency 0.169
and confidence 0.669

Rule 1* ($m = 1.0$):
if var heart frequency ≤ 110.16 **and**
var CVP ≥ 7.00 **and**
var pH ≥ 7.38 **and**
var temperature in $(37.58, 36.78)$ **and**
var leukocytes ≤ 19.45 **and**
var haematocrit ≥ 28.44 **and**
var calcium ≤ 2.50
then class survived **with** frequency 0.077
and confidence 0.627

Another rule example (rule 2 resp. 2*) for class deceased show the case that no test data sample is implied by a core rule with membership degree $m = 1.0$, i.e. frequency $= 0$. This happens every time the rule defining hyper rectangle is statistically too small. Core rule 2* demonstrates that it is not reliable to model too small regions of higher membership degrees. In this case only zero confidence could be attributed for rule 2*.

Rule 2 ($m = 0.3$):
if var systolic blood pressure ≥ 110.80 **and**
var CVP ≥ 4.50 **and**
var leukocytes ≥ 16.72 **and**
var sodium ≤ 146.28 **and**
var calcium ≤ 2.19
then class deceased **with** frequency 0.047
and confidence 0.512

Rule 2* ($m = 1.0$):
if var systolic blood pressure ≥ 139.20 **and**
var CVP ≥ 15.00 **and**
var leukocytes ≥ 23.56 **and**
var sodium ≤ 131.08 **and**
var calcium ≤ 1.89
then class deceased **with** frequency 0.000
and confidence 0.000

It is possible to generate fuzzy antecedents out of the trapezoids, but this topic we will not discuss here. With the help of such classification rules we have recently built an internet based alarm system to warn physicians whenever a septic shock patient becomes very critical [8].

RECOMMENDATION

As we demonstrated triangle membership functions and trapezoid membership functions with too small core regions are not appropriate for modeling regions (resp. a -rules) where the rule confidence should be very high both for training and test data. Of course, the solution for this dilemma is not to allow the system to generate too small core regions. Since the core region is a single point when using the triangle membership function, it represents the most confident data region in the worst possible way. The region with the highest membership degree represents usually not the region with the highest rule confidence on real world data. Larger regions are statistically more appropriate to model regions of a high membership degree. Therefore, we recommend the use of trapezoid membership functions with larger core regions rather than trapezoid membership functions with small core regions or even triangle membership functions where the core region is a single point only.

CONCLUSION

We discussed the usage of triangle and trapezoid membership functions. Both types of functions are reliable in practice. The main disadvantage using membership functions with too small core regions (e.g. triangle functions) considering applications in practice (real world data) is the less rule confidence of the a -regions resp. a -rules of higher membership degrees a. In fact, for membership degree 1 the corresponding rule confidence could be the worst. Thus, the high membership degree is misleading with respect to rule confidence. By different examples we showed that it is more reliable to use trapezoid functions, avoiding too small core regions. For every dataset and every rule exists an individual best, not too small size of the core region. Finding the optimal size of core regions of membership functions by calculation or adaptation is still an open problem.

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